1. A rod $AB$ of mass $m$ and length $L$ may slide without friction on a horizontal surface. Another rod $CD$, also of mass $m$ and length $L$, is attached at its center to end $B$ of rod $AB$ by a frictionless pin and a torsional spring. The spring has torsional stiffness $k$ and is undeformed when $\theta = 0$. In the figure, $(x_1, y_1)$ and $(x_2, y_2)$ are the locations of the centers of mass of rods $AB$ and $CD$, respectively. A) How many degrees of freedom does the system have? B) Choose appropriate generalized coordinates and derive the equations of motion for the system using Lagrange’s equations. Remember that using constrained generalized coordinates may lead to simpler equations. C) Use your equations to identify any constants of motion that exist for the system, and interpret these in physical terms.

2. The system in (1) is now modified so that end $A$ of rod $AB$ is fixed, i.e. rod $AB$ can move only by rotating around point $A$. A) How many degrees of freedom does the system have now? B) Choose appropriate generalized coordinates and derive Lagrange’s equations of motion for the system.
3. The system in (1) is modified again by attaching a knife edge (skate) to the bottom of rod \(AB\) at end \(A\). (See figures below.) The knife edge engages the horizontal surface and prevents end \(A\) from moving in a direction perpendicular to the rod. In other words, at any given instant, the velocity of \(A\) will be in the direction of the length of the rod as shown in Figure 2a. A) How many degrees of freedom does the system have now? B) Choose appropriate generalized coordinates and derive the equations of motion for the system using Lagrange’s equations.