Problem 1
Let \( G(s) \) denote a model for an atomic force microscope. Three separate controllers \( K_i(s) \) \( (i = A, B, C) \) have been designed for the following classical feedback system below.

The goals are to track references, reject disturbances and be insensitive to sensor noise. The sensor noise \( n \) is most important in the bandwidth of \( 10^6 \) to \( 10^{10} \) radians/sec. The disturbances are most important in the bandwidth of \( 10^2 \) to \( 10^4 \) radians/sec. The frequency responses of the loop \( L_i(s) = G(s)K_i(s) \) for each of the three controllers is shown on the next page. Which control design would you select? Explain the reasoning for your choice.
Problem 2
Consider the following cascaded system:

\[ r \xrightarrow{G_1(s)} G_2(s) \xrightarrow{} y \]

Assume \( r(t) \) is a unit step input and sketch the output response \( y(t) \) vs. \( t \) for the following pairs of transfer functions:

A. \( G_1(s) = \frac{1}{10s+1} \) and \( G_2(s) = \frac{100}{3s^2+16s+100} \)

B. \( G_1(s) = \frac{100}{3s^2+16s+100} \) and \( G_2(s) = \frac{1}{10s+1} \)
Problem 3
Consider an oscillator with nonlinear restoring force $\ddot{y} + 2\dot{y} + 5y + y^2 = 0$, or

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ -x_1^2(t) \end{bmatrix}.$$

A. Find all equilibrium points for the oscillator system.
B. Determine the linearized dynamics about each equilibrium point. Report your result as a state-space system description.
C. Sketch the phase-portrait for the linearized dynamics about each equilibrium point. Highlight any invariant directions and specify which one(s) are slow versus fast. Draw arrows on trajectories to show the direction of evolution.
D. Sketch an approximate phase-portrait for the original nonlinear oscillator. Describe (in words) the influence of initial conditions on the resulting state trajectories.
Problem 4
Consider the classical feedback system shown below with \( G(s) = \frac{1}{s-1} \) and \( K(s) = \frac{3.8s+4}{s} \)

A. What is the closed-loop transfer function from \( d \) to \( y \)? What are the poles of this transfer function?
B. A Bode plot of the loop $L(s) = G(s)K(s)$ is provided below. What is the gain margin for the system? What is the time delay margin?
Problem 5
Consider the communication system shown.

A binary message (sequence of ones and zeros) is first encoded, then transmitted by a communication channel, and finally decoded to obtain the received message. Assume that the channel transmits the signal \( w \) without noise and that the encoder algorithm is described by

\[
w(t) = u(t) + u(t - 1) + u(t - 2), \quad t \in \mathbb{N}
\]

where we set \( u(t) = 0 \) for \( t < 0 \).

A. Determine a linear time-invariant state-space description and appropriate initial state \( x(0) = x_0 \) such that the input-output relation is identical with (1). Verify this for the input sequence \( (0 1 1 0 1) \).
B. Design a decoder algorithm that uses the encoded channel signal $w(t)$ to reconstruct a time-shifted version of the input message (i.e., the output message from the decoder will be $y(t) = u(t-1)$). Report the decoder algorithm in the form of a linear time-invariant state-space system with an appropriate initial condition $\bar{x}(0) = \bar{x}_0$. 

C. Use your response to (A) and (B) to form a full-order state-space description of the complete communication system, consisting of the encoder and the decoder coupled in series.
D. Find a minimal state-space realization for the communication system. Justify your response. (Note: you do not need to use the results from previous steps to complete this part of the problem.)
Problem 6

Consider a stable, linear time-invariant system with transfer function $G(s)$. Assume that $G(0) > 0$. Show that for some $K > 0$ the closed-loop system below is stable.