Optimal sensor and actuator selection in dynamic networks

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MOTIVATION

Select **optimal subset** of potential sensors/actuators

- Sensor/Actuator types
- Sensor/Actuator locations

Applications

- Heterogeneous robotic networks
- Phasor Measurement Units in power networks
- Sensors and actuators in flexible aircraft wings

ACTUATOR SELECTION

MODEL

Linear system with many actuators

$$x \;=\; A\,x \;+\; B_1\,d \;+\; B_2\,u$$

PERFORMANCE MEASURE

Steady-state variance amplification

$$\lim_{t} \;\mathcal{E}\;\; x^T(t)\,Q\,x(t) \;+\; u^T(t)\,R\,u(t)$$

OBJECTIVE

Identify **row-sparse state-feedback controller**

$$u \;=\; \succ K x$$

to balance:

PERFORMANCE: **variance amplification**

SPARSITY: **number of actuators**

OPTIMIZATION PROBLEM

$$\text{minimize} \quad J(K) \quad + \quad \sum_{i=1}^{m} \; \mathrm{e}_i^T K \;{}_2$$

variance amplification — **sparsity-promoting penalty function**

$> 0 \;\succ$ variance amplification vs sparsity tradeoff

CONVEX FORMULATION

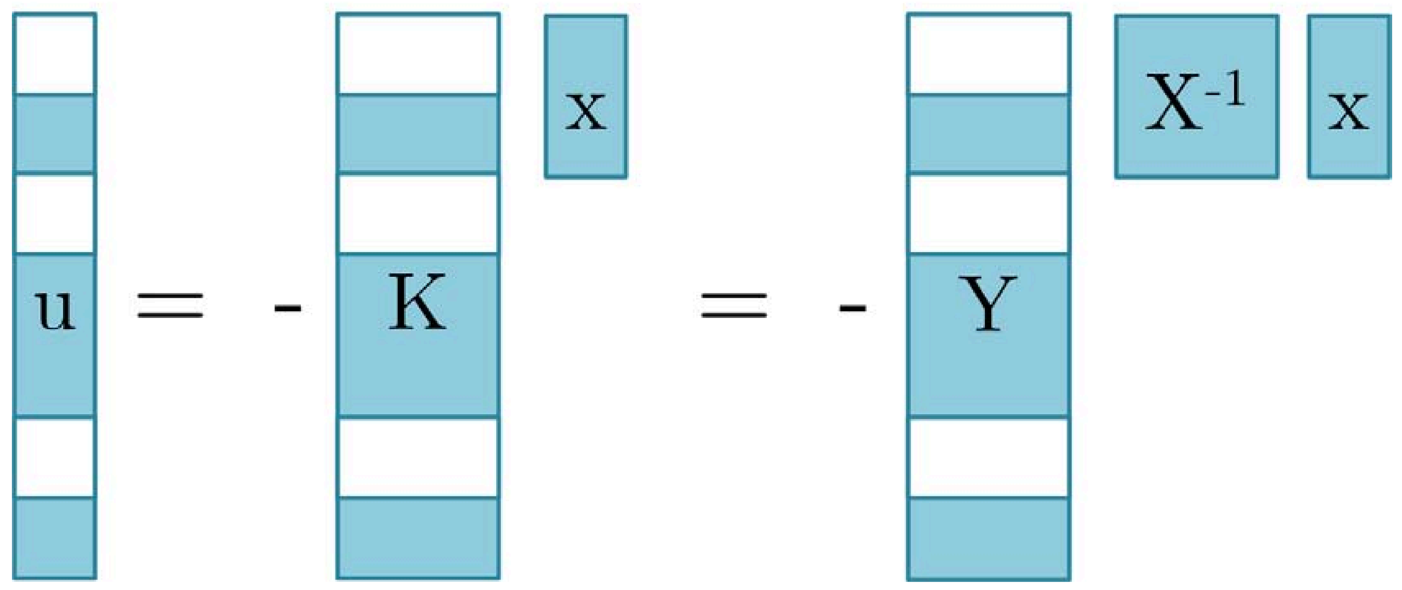
CHALLENGE

$J(K)$ – non-convex function of K

KEY OBSERVATION

Change of variables $Y \;:=\; K X$

- Yields convex dependence of $J(K)$ on X and Y
- Preserves row-sparse structure



SEMIDEFINITE PROGRAM

$$\text{minimize} \quad J(X\;Y) \;+\; \sum_{i=1}^{m} \; \mathrm{e}_i^T Y \;{}_2$$

$$\text{subject to} \quad AX + XA^T \succ B_2 Y \succ Y^T B_2^T + B_1 B_1^T = 0$$

$$X \quad 0$$

EFFICIENT ALGORITHM

ALTERNATING DIRECTION METHOD OF MULTIPLIERS

Form augmented Lagrangian

$$\mathcal{L}\;(X\;Y\;\;) \;:=\; J(X\;Y) \;+\;\; g(Y) \;+\; \phi\;(X\;Y\;\;)$$

by dualizing and penalizing linear constraint, $h(X\;Y)$,

$$\phi\;(X\;Y\;\;) \;:=\; \text{trace} \;\;{}^T h(X\;Y) \;\;+\; \frac{}{2}\;\; h(X\;Y)\;{}_F^2$$

ITERATIVELY SOLVE TRACTABLE SUBPROBLEMS

$$X_{k+1} \;=\; \underset{X}{\arg\min}\;\; \mathcal{L}\;(X\;\;Y_k\;\;\;{}_k) \qquad \textit{Projected Descent}$$

$$Y_{k+1} \;=\; \underset{Y}{\arg\min}\;\; \mathcal{L}\;(X_{k+1}\;\;Y\;\;\;{}_k) \qquad \textit{group LASSO}$$

$${}_{k+1} \;=\; {}_k \;+\;\; h(X_{k+1}\;\;Y_{k+1})$$

SENSOR SELECTION

ESTIMATE STATE x FROM NOISY OUTPUT y

$$x \;=\; A\,x \;+\; B_1\,d$$
$$y \;=\; C\,x \;+$$

Identify **observer gain** to balance

PERFORMANCE: **variance amplification**

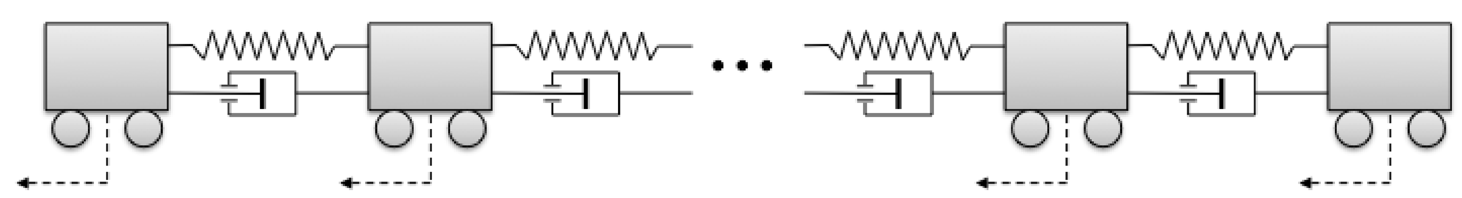
SPARSITY: **number of sensors**

KEY POINT

Can be brought to actuator selection problem

A SENSOR SELECTION EXAMPLE

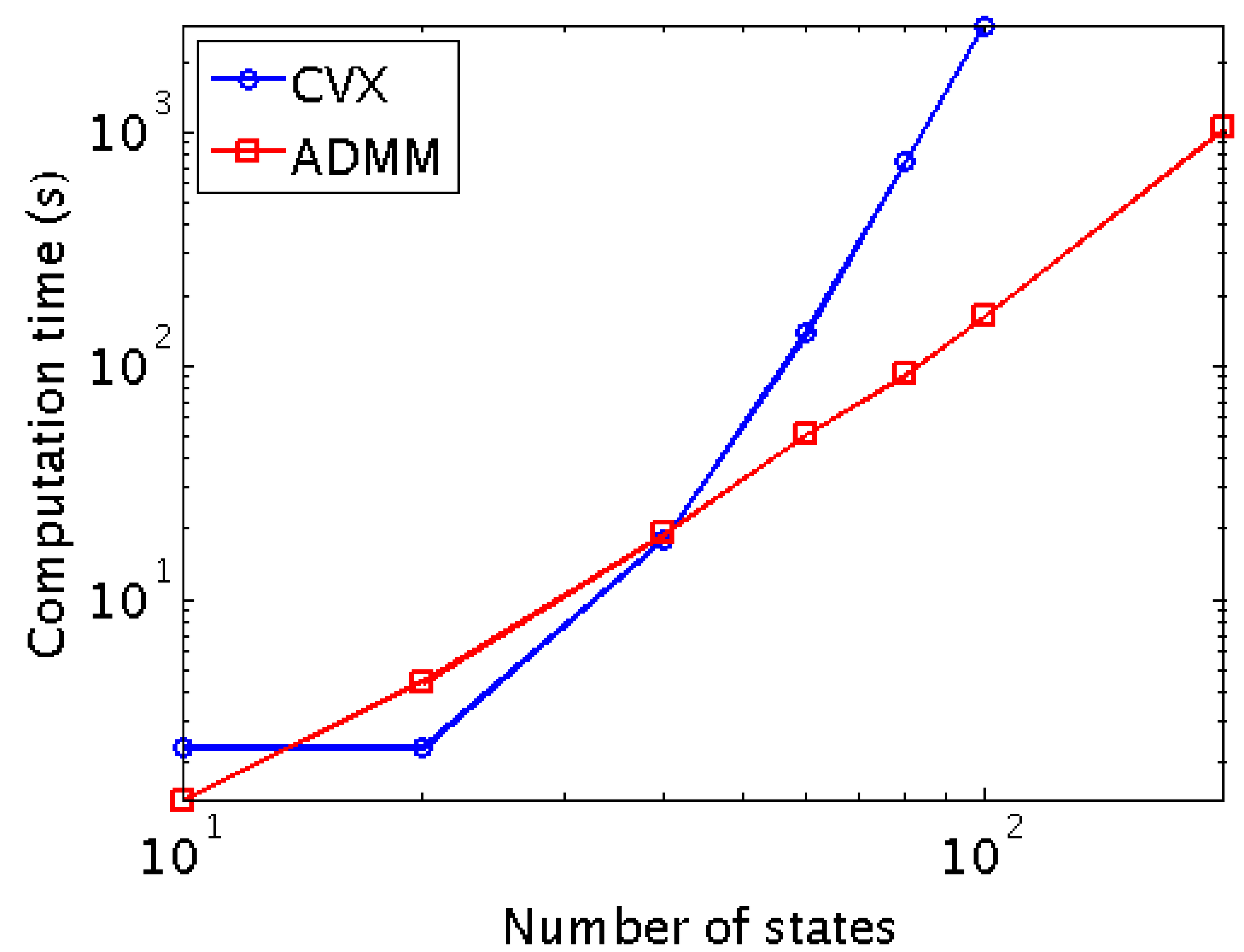
VEHICULAR FORMATION



OBJECTIVE

Optimal GPS placement

CVX vs ADMM for $\;= 100$:



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