Optimal sensor and actuator selection in dynamic networks

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**Motivation**

Select optimal subset of potential sensors/actuators
- Sensor/Actuator types
- Sensor/Actuator locations

Applications
- Heterogeneous robotic networks
- Phasor Measurement Units in power networks
- Sensors and actuators in flexible aircraft wings

**Actuator Selection**

**Model**

Linear system with many actuators
\[ x = Ax + B_1 d + B_2 u \]

**Performance Measure**

Steady-state variance amplification
\[ \lim_{t \to \infty} E \left( x^T(t) Q x(t) + u^T(t) R u(t) \right) \]

**Objective**

Identify row-sparse state-feedback controller
\[ u = \lambda K x \]

to balance:

**Performance:** variance amplification

**Sparsity:** number of actuators

**Optimization Problem**

minimize \[ J(K) + \sum_{i=1}^{m} e_i^T K e_i \]

subject to \[ AX + XA^T \succ B_2 Y \succ Y^T B_2^T + B_1 B_1^T = 0 \]

\[ X \succeq 0 \]

**Efficient Algorithm**

**Alternating Direction Method of Multipliers**

Form augmented Lagrangian
\[ \mathcal{L}(X, Y) := J(X, Y) + g(Y) + \phi(X, Y) \]

by dualizing and penalizing linear constraint, \( h(X, Y) \),

\[ \phi(X, Y) := \text{trace}(X Y) + \frac{1}{2} h(X, Y) \]

**Iteratively Solve Tractable Subproblems**

\[ X_{k+1} = \arg\min_X \mathcal{L}(X, Y_k, k) \text{ Projected Descent} \]

\[ Y_{k+1} = \arg\min_Y \mathcal{L}(X_{k+1}, Y, k) \text{ group LASSO} \]

\[ k_{k+1} = k + h(X_{k+1}, Y_{k+1}) \]

**Sensor Selection**

**Estimate state \( x \) from noisy output \( y \)**

\[ x = Ax + B_1 d \]

\[ y = C x + \]

Identify observer gain to balance

**Performance:** variance amplification

**Sparsity:** number of sensors

**Key Point**

Can be brought to actuator selection problem

**A Sensor Selection Example**

**Vehicular formation**

Objective

Optimal GPS placement

CVX vs ADMM for \( n = 100 \):

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